Application of Bayesian Chain Ladder Models in Prediction Range Reserves of Motor Vehicle Insurance Claims in Indonesia (Case Study of XYZ Insurance Companies)

[1]Zaky Nurzamzami, [2]Bona Christanto Siahaan

Abstract— The main problem often faced by insurance companies is estimating claim reserve. The calculation of claim reserve that is undertaken inaccurately will affect the business operations of the insurance company. The claim reserve estimation method that is commonly undertaken called Chain Ladder method and its variations. Besides, Peters, Targino, and Wuthrich (2017) develop a method namely gamma-gamma Bayesian Chain Ladder. This is a Bayesian Chain Ladder method that uses a gamma distribution and has the prediction range of claim reserve that relatively small. The main purpose of this research is implementing the process of calculation prediction range with the gamma-gamma Bayesian Chain Ladder model in the context XYZ insurance companies in Indonesia, and compare it with the Chain Ladder (Mack's model). The data used in this research is product claim of vehicle insurance company XYZ data from 2014 to 2016. The results of the prediction range based on the MSEP value of the gamma-gamma Bayesian Chain Ladder model from 2014 to 2016 relatively smaller compared to the Mack's MSEP Chain Ladder model.

Index Terms — Claim Reserving, Chain Ladder, Gamma-Gamma Bayesian Chain Ladder

1 Introduction

Claim reserves are amounts of money that will be prepared by the insurance company to make future payments related to claims that have already occurred but have not been paid or settled at a certain period (Maher, 1992). Settlement of claim payments at insurance companies is usually carried out after being reported. But in some insurance products, settlement of claim payments requires a long time or delayed payment for certain periods (Wuthrich and Merz, 2008). According to Hossack, Pollar, and Zenwirth (1999) explain there are two types of claim reserves, namely Incurred But Not Reported (IBNR) and Reported But Not Settled (RBNS). Incurred But Not Reported is defined as an event that has occurred, but has not been reported to the insurance company. While Reported But Not Settled is defined as an event that has been reported but the payment has not been resolved.

Some statistical models that can be used in estimating claims reserves are generally divided into two, namely methods that are deterministic and stochastic. Conceptually, deterministic trends are related to functions that are not random from time, while the stochastic trends is related to functions that are random and change over time. The Chain Ladder method is categorized as a deterministic method which can not be modeled when there are variations in the data. Initially, the Chain Ladder method was introduced in a purely algorithmic and it was not based on a stochastic model. Then stochastic methods include frequentist and Bayesian methods.

The Chain Ladder technique stochastic model was first popularized by Kremer (1982), then a development was develop by Mack (1993) and Englan and Verral (2002). Taylor (2015) explains that the Bayesian Chain Ladder (BCL) model is a combination of stochastic and deterministic methods, allowing for more accurate estimates. Peters, Targino, and Wuthrich (2017) explain one of the advantages of the Bayesian model framework, which allows closed form solutions. In addition, it also explains the combination of Bayesian models based on the stochastic method with the Chain Ladder model based on deterministic methods allowing estimation that has a sensitivity level to results with a smaller error rate. England and Verral (2002) explain the combination of Chain Ladder Method that is not paying attention to the existence of past information in its calculations which causes the estimation results are less accurate. The Bayesian model itself has the advantage of allowing the use of initial information (priors) in its calculations that are useful in posterior calculations or opportunities sought from data.

This study aims to apply the Bayesian Chain Ladder model in calculating the predicted range of claim reserves in the insurance context in Indonesia. The ability to produce predictive values for error rates based on mean square error of prediction (MSEP) is relatively small. This is done to see whether the Bayesian Chain Ladder model can be used in the context of Indonesia insurance.

2 Literature review

2.1 Claim Reserves

According to Wuthrich and Merz (2008), insurance claims cannot be settled immediately when they occur. There are usually delays in claim reporting and there are delays in resolving claims. As a consequence of the delay, insurance companies need to predict future cash flows from claims that have occurred in the past and are resolved in the future.

2.2 Claim Reserve Analysis Method

In analyzing claims reserves can be done with various methods of estimating claims reserves.

2.2.1 Chain Ladder Method

This method is widely known as one of the actuarial methods in calculating estimated reserves because it is an easy method to implement. In its application, this technique can be applied to cumulative claim values and is specifically intended to predict incremental claim amounts in empy cells from those defined as the upper triangle area of the run-off triangle. The run-off triangle is a top triangular matrix that has symmetrical properties. Therefore, there is a possibility of generalizing a symmetrical triangle geometric arrangement on the lower triangle of the run-off triangle.

Antonio et al. (2006) explain that the run-off triangle data contains an overview of the overall claim (aggregate) and is a summary of a data set of individual claims. Data in the run-off triangle data is usually one of two possibilities, namely the amount of claims or number of claims, both of which are presented in incremental or cumulative forms. Suppose that a declares a random variable the size of the claim (in the form of incremental data) for claims that occur in the accident period i and is paid in the development period j, where $1 \le i \le n$ and $1 \le j \le n$ (Olofsson, 2006).

Run-off triangle data in cumulative form $C_{i,j}$, can be formed based on incremental $D_{i,j}$ with the following calculation formula:

$$C_{i,j} = \sum_{k=1}^{j} D_{i,k} \ ; \ 1 \le i \le n, \ 1 \le j \le n \ \text{and} \ i + j \le n+1 \tag{1}$$

The total loss reserve (R_i) is the sum of all $D_{i,j}$ in the future triangle. The loss reserve needs to be estimated by first estimating the outstanding claims in the future triangle using information from the triangle run-off data. For example $\check{D}_{i,j}$ is an estimator for $D_{i,j}$ in the future triangle, then the loss reserve for the accident period i is estimated by the following formula:

$$R_i = \sum_{k=n+1-i}^{n} \check{D}_{i,k} \ ; \text{for} \ i = 2, \dots, n.$$

Then obtained the total loss reserves with the following formula:

$$R = \sum_{i=2}^{n} \sum_{k=n+1-i}^{n} \check{D}_{i,k} \tag{2}$$

2.2.2 Gamma-Gamma Bayesian Chain Ladder Method

The Bayesian Chain Ladder mdoel can be developed with gamma-gamma estimation, as explained by Wuthrich and Merz (2015) known as the gamma-gamma Bayesian Chain Ladder model. Estimation of claim backup with gamma-gamma Bayesian Chain Ladder is a combination of estimated reserve estimates through the Chain Ladder model then combined with Bayesian models that use gamma distribution. It also introduces the gamma-gamma Bayesian Chain Ladder model that can be used to obtain a closed form solution for conditional mean square error of prediction (MSEP). Peters et al. (2013) added that the gamma-gamma Bayesian Chain Ladder model can produce a mor accurate solution, with a smaller mean square error of prediction.

Wuthrich and Merz (2015) explaining in the development of the gamma-gamma Bayesian Chain Ladder model it can take a Bayesian perspective and enter parameter uncertainty into the stochastic model. The following formulation model assumes for posterior factors:

$$\hat{f}_j^{BCL(t)} = E\left[\theta^{-1} \mid F_t\right] = \omega_j^{(t)} . \hat{f}_j^{(t)} + \left(1 - \omega_j^{(t)}\right) . f_j \tag{3}$$

$\hat{f}_j^{(t)}$ is a Chain Ladder factor, $\omega_j^{(t)}$ is a credibility of weight, and f_j is the estimate of the prior. The following of the credibility of weight $\omega_j^{(t)}$:

$$\omega_j^{(t)} = \frac{\sum_{i=1}^{(t-j-1)\wedge I} C_{i,j}}{\sum_{i=1}^{(t-j-1)\wedge I} C_{i,j} + \sigma_j^2(\gamma_j - 1)} \in (0,1) \tag{4}$$

In the next stage, Wuthrich and Merz (2015) explained that to determine the mean square error of prediction (MSEP) of the gamma-gamma Bayesian Chain Ladder model for uncertainty in short-term predictions are formulated as follwos:

$$msep_{\sum_i CDR_i^{(t-1)} \mid D_t}(0) = \left(\hat{C}_{i,j}^{CL(t)}\right)^2 \left[\frac{\sigma_{t-1}^2}{C_{i,t-1}} + \frac{\sigma_{t-1}^2}{\sum_{l=1}^{i-1} C_{l,t-1}} + \sum_{j=t-i+1}^{j-1} \bar{\sigma}_j^t \frac{\sigma_j^2}{\sum_{l=1}^{i-j-1} C_{l,j}} \right] \tag{5}$$

2.2.3 Mean Square Error of Prediction (MSEP)

Mean square error of prediction (MSEP) is a measurement of the accuracy of estimation which in this case is the accuracy in analyzing the results of the prediction of claims reserves (Wuthrich and Merz, 2008). Lehman and Casella (1998) explain the mean square error (MSE) of an estimator to measure the average of the squared error or deviation which is the difference or deviation between the estimated value and the estimated error squared value of the loss.

In addition, Wuthrich and Merz (2008) describe conditional mean square error of prediction from predictors $\hat{X}$ for X as follows:

$$\begin{aligned} msep_{X|D}\left(\hat{X}\right) &= E\left[\left(X - \hat{X}\right)^2 \mid D\right] \\ &= E\left[\left(\left(X - E(X)\right)^2 + 2\left(X - E(X)\right)\left(E(X) - \hat{X}\right) + \left(E(X) - \hat{X}\right)^2\right) \mid D\right] \\ &= Var\left(X \mid D\right) + \left(E\left(X \mid D\right) - \hat{X}\right)^2 \end{aligned}$$

From the equation, it can be seen that the predictor $E\left(X \mid D\right) = \hat{X}$ will minimize MSEP, so that it is obtained:

$$msep_{X|D}\left(\hat{X}\right) = Var\left(X \mid D\right) \tag{6}$$

Referring to Wuthrich and Merz (2015) mentioning to calculate the one-year uncertainty mean square error of prediction which assumes that the existence of non-informatice priors as defined as follows:

$$msep_{\,CDR_{1,t+1}|D_t}(0) \approx e_{i,t+1}^{(t)} \stackrel{\text{def}}{=} msep_{CDR_{1,t+1}|D_t}^{MW}(0)$$

$$= \left(\hat{C}_{m,j}^{BCL(t)}\right)^2 \left[\frac{\sigma_{t-i}^2}{C_{i,t-i}} + \frac{\sigma_{t-i}^2}{\sum_{l=1}^{i-1} C_{l,t-1}} + \sum_{j=t-1+1}^{j-1} \bar{\sigma}_j^{\,t} \frac{\sigma_j^2}{\sum_{l=1}^{i-1} C_{l,j}}\right] \qquad (7)$$

Lehmann and Casella (1998) and Wuthrich and Merz (2008) explain the root mean square error (RMSE) that RMSE is rated as having a more intuitive scale to MSE that has a scale value in the same measurement of the data being observed.

$$RSME = msep^{1/2} = \sqrt{Var\left(X \mid D\right)} = \sqrt{\sigma^2\left(X \mid D\right)} = \sigma\left(X \mid D\right).$$

3 Methodology

This research was conducted to apply the gamma-gamma Bayesian Chain Ladder method in calculating the predicted range of claim reserves in XYZ insurance companies, then comparing the results of predicted claims reserves in the XYZ insurance company business line between the gamma-gamma Bayesian Chain Ladder method and Chain Ladder method. The research data used is data on motor vehicle insurance claims. Data on motor vehicle insurance claims used are four-wheeled motorized vehicles. Determination of the sample is done by purposive sampling technique. According to Sekarang and Bougie (2013), purposive sampling is a design that is limited to specific samples that can provide the information needed because only those samples that have information or meet the criteria set by the study are sampling techniques with certain considerations. Considerations used in determining the sample include research data which are IBNR claims on motor vehicle insurance for XYZ insurance companies from 2014 to 2016, the researcg data used are data on motor vehicle insurance claims in XYZ insurance companies in the period 2014 to 2016 (January 2014 to December 2016), and data in the monthly period in the context of the motor vehicle insurance business line in Indonesia is a line of business with a short-term period or less than one year.

This research begins with collecting data, namely the reported claims data, the development period where the claim is resolved, and the amount of claims paid which then the data is described according to the type of numeric data or not. Claim data used is the IBNR claim data for four-wheeled motorized vehicles in the XYZ insurance company whose data type is numeric. Furthermore, the formation of data obtained in the form of a triangle run-off. This is because in estimating claim quantities, the method used in this study requires data in the form of a triangle run-off. Establishment of a run-off triangle by separating vehicle insurance data per annual period and calculating the occurrence and payment of claims that occur in each period and for each payment period (lag period). After the data is formed through the run-off triangle, an estimation of the claims is made using two methods, namely the Chain Ladder method and gamma-gamma Bayesian Chain Ladder method. After estimation, mean square error of prediction (MSEP) was calculated as an effort to determine the best model between predictions of errors with the Chain Ladder (Mack's model) and gamma-gamma Bayesian Chain Ladder.

4 Results and Discussion

4.1 Results

4.1.1 Claim Reserves 2014 to 2016

To estimate claim reserves using the gamma-gamma Bayesian Chain Ladder method will have the same value as the Chain Ladder method because in the gamma-gamma Bayesian Chain Ladder method there are non-informative prior assumptions that make values (prior limit that are non-informative) so produce the value of the credibility of the weight and also the value of the gamma-gamma Bayesian Chain Ladder factor will be the same as the Chain Ladder factor, then the next result will be estimated claim reserves using the Chain Ladder method in the table every year from 2014 to 2016.

4.1.1.1 2014 to 2016 Data

After inputting XYZ insurance company claim data in 2014 in the incremental and cumulative run-off triangle and also Chain Ladder factors, the estimation results will be explained in the table below:

Table 1: Results of the 2014 Claims Reservation

Accident Period	*Diag. Payments*	*Ultimate Claim Prediction*	*CL reserves* $\left(R_i^{CL(I)}\right)$
Januari	286.267.384	286.267.384	-
Februari	231.913.675	242.452.188	10.538.513
Maret	216.711.195	231.495.590	14.784.395
April	282.827.950	336.499.418	53.671.468
Mei	240.869.450	302.989.205	62.119.755
Juni	377.311.000	497.737.978	120.426.978
Juli	173.417.285	262.155.505	88.738.220
Agustus	310.073.000	502.028.756	191.955.756
September	487.898.900	881.476.517	393.577.617
Oktober	355.725.885	907.153.755	551.427.870
November	145.812.500	719.932.334	574.119.834
Desember	14.829.000	781.279.622	766.450.622
Total	3.123.657.224	5.951.468.252	2.827.811.028

Table 2: Results of the 2015 Claims Reserves

Accident Period	*Diag. Payments*	*Ultimate Claim Prediction*	*CL reserves* $\left(R_i^{CL(I)}\right)$
Januari	507.440.920	507.440.920	-
Februari	646.114.100	663.582.743	17.468.643
Maret	470.598.620	530.512.929	59.914.309
April	324.422.650	373.300.870	48.878.220
Mei	381.614.075	443.721.628	62.107.553
Juni	466.150.000	559.769.965	93.619.965
Juli	297.666.575	371.507.121	73.840.546
Agustus	397.546.650	549.707.689	152.161.039
September	241.050.345	389.688.548	148.638.203
Oktober	228.953.473	512.025.885	283.072.412
November	178.369.165	690.014.790	511.645.625
Desember	8.953.000	496.248.230	487.295.230
Total	4.148.879.573	6.087.521.319	1.938.641.746

Table 3: Results of the 2016 Claims Reserves

Accident Period	*Diag. Payments*	*Ultimate Claim Prediction*	*CL reserves* $\left(R_i^{CL(I)}\right)$
Januari	554.684.900	554.684.900	-
Februari	471.929.800	492.669.188	20.739.388
Maret	483.483.179	519.221.176	35.737.997
April	902.261.680	1.006.028.689	103.767.009
Mei	388.956.194	460.140.304	71.184.110
Juni	376.744.350	461.293.699	84.549.349
Juli	258.890.551	331.613.724	72.723.173
Agustus	431.550.750	582.897.540	151.346.790
September	545.472.600	786.742.372	241.269.772
Oktober	402.374.480	601.888.719	199.514.239
November	338.574.140	655.144.228	316.570.088
Desember	99.164.420	751.396.744	652.232.324
Total	5.254.087.044	7.203.721.283	1.949.634.239

From table 1 to table 3 shows the amount of claim reserves in the Period February to December of the following year. For example, the total claim reserves obtained from 2014 data amounted to Rp 2,827,811,028, meaning that the insurance company must prepare a claim reserve fund of that value in 2015. Likewise with 2015 and 2016.

4.1.2 MSEP Gamma-Gamma Bayesian Chain Ladder Method and Chain Ladder Method (Mack's Model)

After obtaining the estimated reserve claim value where the estimated claim reserves using the gamma-gamma Bayesian Chain Ladder method will have the same value as the Chain Ladder method, the MSEP is calculated using the gamma-gamma Bayesian Chain Ladder method and Chain Ladder method (Mack's model) 2014 to 2016.

Table 4: Rooted MSEP Results Gamma-Gamma Bayesian Chain Ladder Method and Chain Ladder Method (Mack's Model) in 2014

Accident Period	*Claim Reserves*	$msep^{(1/2)}$ Mack	$msep^{(1/2)}$ BCL
Januari	-	-	-
Februari	10.538.513	41.896	29.660,31
Maret	14.784.395	1.428.762	805.784,18
April	53.671.468	63.866.638	39.516.310,91
Mei	62.119.755	60.907.791	36.091.982,87
Juni	120.426.978	85.375.188	59.509.472,11
Juli	88.738.220	68.286.428	35.084.472,12
Agustus	191.955.756	112.648.584	70.466.166,84
September	393.577.617	185.196.248	132.072.309,65
Oktober	551.427.870	243.880.644	168.635.809,85
November	574.119.834	286.231.577	197.933.799,56
Desember	766.450.622	580.243.101	860.806.681,76
Total	2.827.811.028	1.688.106.858	1.600.952.450,13

Table 4 explains that the MSEP value of 2014 data with the gamma-gamma Bayesian Chain Ladder method has a smaller value compared to the MSEP Chain Ladder method (Mack's model). In addition, it can also be said that the MSEP value of the gamma-gamma Bayesian Chain Ladder method and Chain Ladder method (Mack's model) has a relatively increasing value in each period.

Table 5: Rooted MSEP Results Gamma-Gamma Bayesian Chain Ladder Method and Chain Ladder Method (Mack's Model) in 2015

Accident Period	*Claim Reserves*	$msep^{(1/2)}$ Mack	$msep^{(1/2)}$ BCL
Januari	-	-	-
Februari	17.468.643	25.001.821	19.330.594,60
Maret	59.914.309	83.166.103	51.773.764,22
April	48.878.220	67.894.140	36.974.488,46
Mei	62.107.553	75.934.859	44.009.762,07
Juni	93.619.965	90.283.055	56.115.295,08
Juli	73.840.546	70.357.331	37.451.670,61
Agustus	152.161.039	113.198.577	62.162.041,86
September	148.638.203	111.728.578	49.421.763,93
Oktober	283.072.412	173.818.662	82.928.503,79
November	511.645.625	320.311.989	190.035.617,97
Desember	487.295.230	359.901.614	578.215.133,02
Total	1.938.641.746	1.491.596.728	1.208.418.635,60

Table 5: Rooted MSEP Results Gamma-Gamma Bayesian Chain Ladder Method and Chain Ladder Method (Mack's Model) in 2016

Accident Period	*Claim Reserves*	$msep^{(1/2)}$ Mack	$msep^{(1/2)}$ BCL
Januari	-	-	-
Februari	20.739.388	5.059.346	3.622.476,69
Maret	35.737.997	12.860.725	7.930.259,97
April	103.767.009	21.835.360	16.024.809,01
Mei	71.184.110	38.066.792	16.561.823,88
Juni	84.549.349	41.039.544	17.560.501,11
Juli	72.723.173	37.007.815	13.408.422,01
Agustus	151.346.790	62.013.405	27.119.848,37
September	241.269.772	83.054.833	39.764.466,25
Oktober	199.514.239	75.773.106	31.717.423,38
November	316.570.088	278.952.662	116.726.159,24
Desember	652.232.324	356.404.503	291.264.210,87
Total	1.949.634.239	1.012.068.091	581.700.400,77

Same as table 4, table 5 and table 6 explains that the MSEP value of the data with the gamma-gamma Bayesian Chain Ladder method has a smaller value compared to the MSEP Chain Ladder method (Mack's model). In addition, it can also be said that the MSEP value of the gamma-gamma Bayesian Chain Ladder method and Chain Ladder method (Mack's model) has a relatively increasing value in each period.

4.1.3 Prediction Range

This section will describe the predicted value of claims reserve per period according to the 2014 to 2016 MSEP Chain Ladder (Mack's model) and gamma-gamma Bayesian Chain Ladder data, as well as actual data on motor vehicle insurance claims for XYZ insurance companies year 2015 to 2017 which will be shown in graphical form.

After obtaining the MSEP in the claim data in 2014 to 2015, the predicted range values of the gamma-gamma Bayesian Chain Ladder method and Chain Ladder (Mack's model) method for claim reserves $\pm\, 2msep^{1/2}$ and actual data claims from 2015 to 2017 will also be displayed.

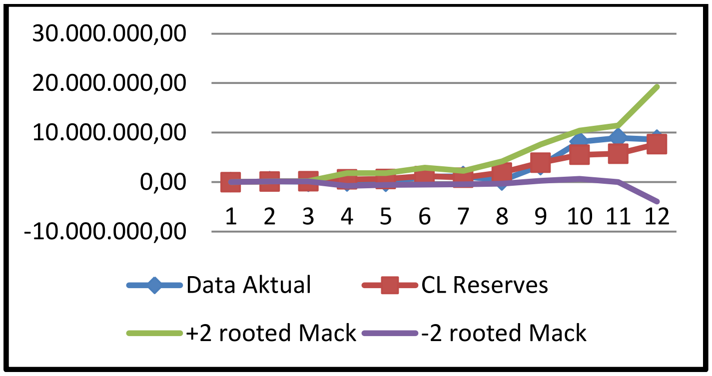

Figure 1: Chart of Prediction of 2014 Chain Ladder Method (Mack's Model) and 2015 Actual Data.

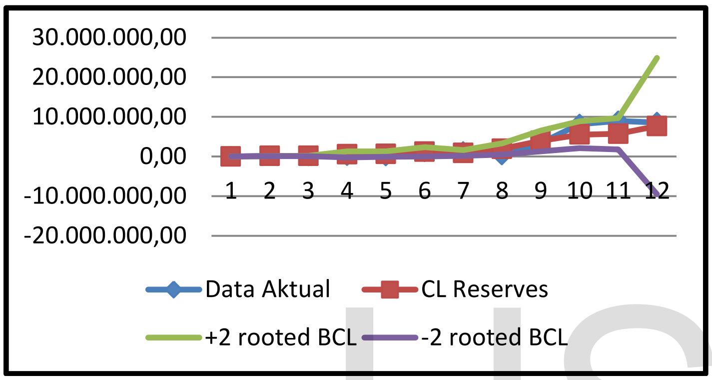

Figure 2: Chart of Prediction of 2014 Gamma-Gamma Bayesian Chain Ladder Method and 2015 Actual Data.

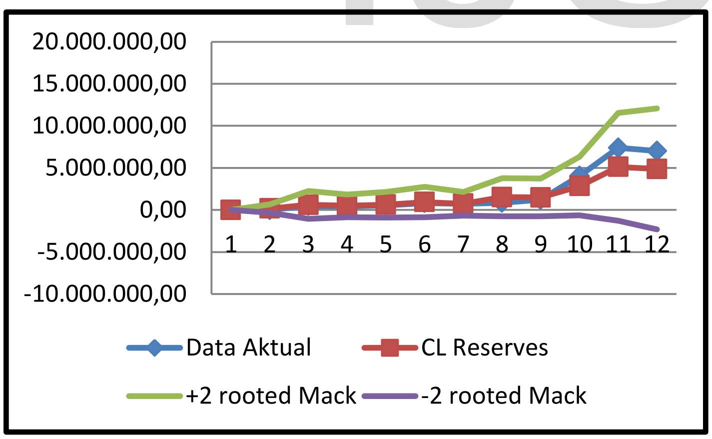

Figure 3: Chart of Prediction of 2015 Chain Ladder Method (Mack's Model) and 2016 Actual Data.

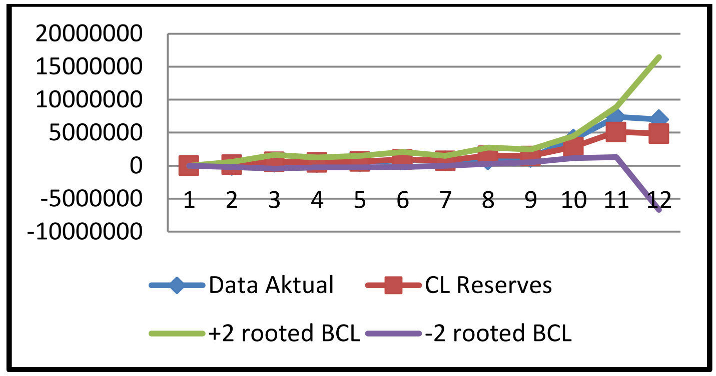

Figure 4: Chart of Prediction of 2015 Gamma-Gamma Bayesian Chain Ladder Method and 2016 Actual Data.

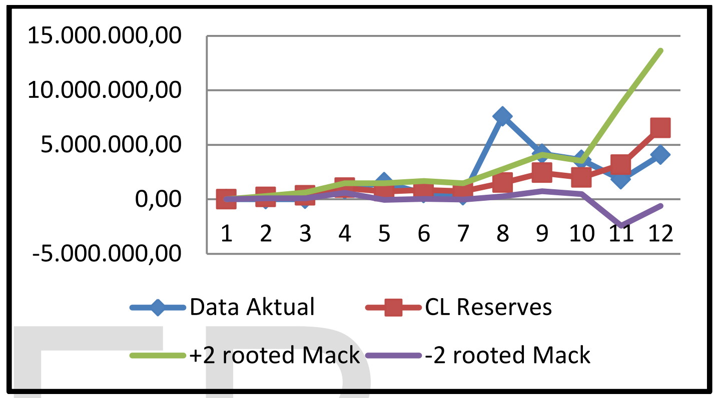

Figure 5: Chart of Prediction of 2016 Chain Ladder Method (Mack's Model) and 2017 Actual Data.

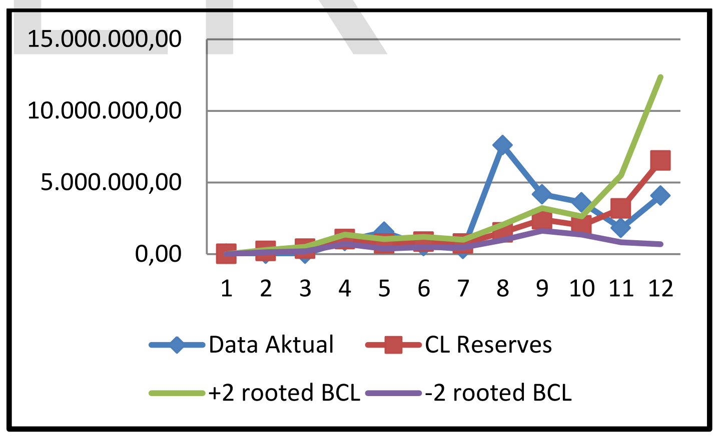

Figure 6: Chart of Prediction of 2016 Gamma-Gamma Bayesian Chain Ladder Method and 2017 Actual Data.

Based on figure 1 to figure 6 explain that the predicted range is calculated at claim reserves $\pm 2msep^{1/2}$. From the predicted range, which method can be obtained which will be closer to the actual data of XYZ insurance company. Gamma-gamma Bayesian Chain Ladder method seen from the graph, the actual data is closer to the prediction range with the gamma-gamma Bayesian Chain Ladder method.

5 Conclusion

The gamma-gamma Bayesian Chain Ladder model can be used as as one method in calculating the predicted range of claims for insurance companies that have a motor vehicle insurance business line.

The results of the prediction range are based on mean square error of prediction (MSEP) with the gamma-gamma Bayesian Chain Ladder model relatively smaller than the MSEP Chain Ladder (Mack's model) both through 2014 to 2016 claim data. In this case the predicted range value uses the gamma-gamma Bayesian Chain Ladder model from 2014 to 2016 in a row Rp 1,600,952,450.12, Rp 1,208,418,635.60, and Rp 581,700,400.77, while the predicted range value uses Chain Ladder (Mack's model) from 2014 to 2016 respectively Rp 1,688,106,858, Rp 1,491,596,728, and Rp 1,012,068,091.

Based on claims data from 2014 to 2016, the estimated reserve value produced by the gamma-gamma Bayesian Chain Ladder model and Chain Ladder model is similar because gamma-gamma Bayesian Chain Ladder model has an assumption of non-informative priors, so that the Bayesian Chain Ladder factor will be worth the same as the Chain Ladder factor and produce the same estimated reserve claim.

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